Consumption Dynamics During the Great Recession

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Abstract

The decline in durable expenditures was a central feature of the Great Recession, yet business cycle models typically abstract from durable consumption or assume that it is costless for households to adjust their durable holdings. We construct a heterogeneous agent business cycle model with fixed costs of durable adjustment and show that this model better matches consumption dynamics over the business cycle than models with costless durable adjustment. Furthermore, fixed costs of durable adjustment induce impulse response functions (IRFs) that are non-linear, history dependent and procyclical. Our estimates imply that during the Great Recession, durable expenditures were half as responsive to additional stimulus as during the 1990s. The procyclical IRF in the model is driven by changes in the distribution of households’ desired durable holdings over the cycle. We test our model by estimating this distribution empirically with PSID microdata and find that it changes cyclically as quantitatively predicted by our model. Finally, we show that our model’s time-varying IRF is consistent with aggregate time-series evidence, in contrast to models with costless durable adjustment.

JEL Classification: E21, E32, D91

Keywords: Durables, Fixed Costs, Consumption, Non-linear Impulse Response

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1 Introduction

The business cycle patterns of durable expenditures are markedly different from those of non-durable expenditures. Figure 1 displays the decline in durable expenditures (including residential investment), non-durable expenditures and GDP during the Great Recession which began in 2007. The decline in durable expenditures was four times larger than the decline in non-durable expenditures. Overall, the decline in durable expenditures accounted for more nearly half of the decline in GDP in

1 We define durable expenditures as NIPA durable expenditures + residential investment. The BEA treats durable and residential investment differently, including housing services in GDP while excluding durable services. In both our model and data analysis, we define GDP as the sum of non-durable expenditures excluding housing services, consumer durable expenditures, and private domestic investment. See Appendix 1.

2 All series are h-pfiltered with smoothing parameter 1600.

3 See Petev, Pistaferri, and Eksten [2012] for additional discussion of the behavior of consumption during the Great Recession.
the most recent recession. This large volatility of durable expenditures is a general feature of U.S. business cycles\(^4\): from 1960-2010 the standard deviation of durable expenditures relative to GDP is 3.02 while the standard deviation of non-durable expenditures to GDP is 0.55. Thus, explaining the behavior of durable expenditures is essential for explaining business cycles. In particular, Leamer [2007] argues that "housing IS the business cycle".

Despite the importance of durables in accounting for output fluctuations, durables are rarely included in standard business cycle models leaving open the question whether standard models can match these facts. In this paper, we build a heterogeneous agent business cycle model with fixed costs of durable adjustment and show that this model is more successful at matching consumption dynamics than are simpler models that assume frictionless durable adjustment. In particular, we argue that our model with fixed costs better matches consumption dynamics along several dimensions: (1) It provides a better fit for the average business cycle volatility of durable and non-durable expenditures. (2) It better match the significant history dependent sensitivity of durable expenditures to aggregate shocks that we document using time-series data. (3) It better matches the behavior of household durable purchases observed in microdata.

We begin by first exploring business cycle models with no costs of durable adjustment. We show that these models generate durable expenditures that are much too volatile and non-durable expenditures that are much too smooth. In addition, we simulate the Great Recession in these models and find that the simulated percentage decline in durable expenditures is an order of magnitude too large. At the same time, the decline in non-durable expenditures in the model is only two thirds of that observed in the data.

Durable expenditures are extremely volatile in these models due to standard stock-flow considerations: households’ utility depends on the stock of durables, and since durables do not depreciate fully from period to period, large decreases in durable expenditure can be consistent with only small decreases in the durable stock.\(^5\) At the same time, non-durable consumption is very smooth because

\(^4\)The following statistics may appear slightly different from standard business cycle statistics. This again is because, unlike the BEA, we treat consumer durables and housing symmetrically in our GDP calculations. Overall our conclusions are not sensitive to this treatment of the data.

\(^5\)This is the same mechanism that generates amplification of investment volatility relative to GDP in the standard RBC model. See e.g. King and Rebelo [1999]. For example, in the data, the real value of the durable stock is nearly 10 trillion dollars. A 1% reduction in non-durable consumption and the durable stock must lead to a decline in durable
the large stock of wealth in the U.S. economy relative to average non-durable expenditures allows for lots of consumption smoothing. In addition, in the models, households want to maintain a constant ratio between their non-durable consumption and their durable stock. This implies that most extensions of the model that dampen the volatility of durable expenditures also dampen the volatility of non-durable expenditures. Thus, an improvement in the model fit along one dimension of consumption leads to a worsening along the other. In order to match the behavior of consumption over the business cycle, we must look for a mechanism that amplifies non-durable volatility while dampening durable volatility.

While a standard fix that improves the model’s performance along this dimension is the addition of quadratic adjustment costs, these adjustment costs are inconsistent with the substantial degree of lumpiness observed in durable purchases using microdata. Furthermore, matching this micro evidence is crucial for matching the history dependent response of durable expenditures to aggregate shocks over the business cycle. In particular, we provide aggregate time-series evidence that shows the sensitivity of aggregate durable expenditures to shocks is varies over the business cycle: after long expansions, durable expenditures are substantially more responsive to aggregate shocks than after recessions. Thus, while quadratic adjustment costs can help the model fit along the volatility dimension, they do not help the model match this time-series evidence and are thus unlikely to be useful for policy analysis over the business cycle.

Instead, we introduce more realistic fixed costs of durable adjustment into the model and show that they dramatically improve the model fit along all dimensions. Unsurprisingly, durable adjustment costs dampen the volatility of durable expenditures, although generating the proper degree of dampening could have required unrealistically large adjustment costs. What is perhaps less obvious is that, while the addition of durable adjustment costs dampens durable volatility, it amplifies the volatility of non-durable expenditures. This is because we calibrate our model to match the fraction of household wealth held in durables, and when we add fixed costs of durable adjustment, this portion of wealth becomes illiquid. It is no longer optimal for households to frequently adjust their durable holdings in order to smooth non-durable consumption. Holding aggregate wealth constant investment of 100 billion dollars, or roughly a 25% reduction in quarterly durable investment relative to an average rate of around 400 billion dollars.
while increasing the fraction of that wealth that is illiquid means that more households are effectively borrowing constrained. These households are similar to the "wealthy hand-to-mouth" consumers generated in Kaplan and Violante [2011]. In their model, households can hold a liquid low-return asset or a high-return illiquid asset that carries a transaction cost. While they associate this illiquid asset with e.g., housing, they do not model the particulars that generate these higher returns. In contrast, since our model explicitly models illiquid durables, we generate additional predictions about the dynamic response of durables to shocks. Our model implies strong interactions between the cross-sectional distribution of household durable holdings and the response to aggregate shocks across time.

Fixed costs of adjustment introduce non-linearities into household durable purchase decisions, and these non-linearities do not disappear with aggregation. This implies that aggregate durable expenditures exhibit non-linear, history dependent impulse responses to aggregate shocks. After a long expansion, pent-up durable demand rises, which means that many households are on the verge of adjusting their durable holdings. This makes aggregate durable expenditures more sensitive to aggregate shocks than after a recession, when fewer households are near the margin of durable adjustment.

The magnitude of the time-varying impulse response is quantitatively large. We simulate the impulse response of durable expenditures at the end of the Great Recession and compare it to the impulse response at the end of 1999. We find that the impulse response during the Great Recession is less than half the response to the same shock during the 90s boom. This has important policy implications since the impulse response computed from a linear VAR will substantially understate the true response during booms and substantially overstate the response during recessions. Given the importance of this time-varying impulse response, we search for additional evidence of this model mechanism, both using household microdata as well as using aggregate time-series data.

Mechanically, time $t$ aggregate durable expenditures are given by
\[
X_t = \int (d_{i,t} - d^*_i) h_t \left( (d_{i,t} - d^*_i) \right)
\]
where the "durable gap" $(d_{i,t} - d^*_i)$ is the difference between household $i$'s current durable stock,
Figure 2: Distributions and Hazards Model Vs. PSID

The top panel of Figure 2 shows how this distribution and hazard move over time in our benchmark model. The time-varying impulse response in our model is driven by variation over the business cycle in the cross-sectional distribution of household durable gaps and adjustment hazards, and so we attempt to directly test this implication by estimating the distribution of gaps and hazards using PSID microdata. The obvious complication with this procedure is that $d_{i,t}$ is unobserved in the data. However, we can exploit restrictions from the structural model to generate a mapping between variables that we do observe in the data and $d^s_{i,t}$. This allows us to use PSID data to estimate the distribution of gaps as well as the empirical adjustment hazard as a function of these gaps to directly test our model. The bottom panel of Figure 2 shows that the resulting estimates using PSID data line up remarkably well with our model.

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8For simplicity, we are ignoring maintenance at the moment.
Finally, we also look for additional evidence of time-varying impulse responses using aggregate time-series data. In the model, durable expenditures become more responsive to shocks during booms than during recessions. This implies that the model with fixed costs exhibits conditional heteroscedasticity: aggregate durable expenditures become more volatile in booms, as the sensitivity of expenditures to shocks rises. In contrast, models with flexible durable adjustment do not imply similar time varying sensitivity. As in Bachmann, Caballero, and Engel [2010], we test this implication using time-series data on aggregate durable expenditures. Consistent with the model, we find that the sensitivity of durable expenditures to shocks is substantially larger during booms than during recessions.

Thus, in addition to providing a better fit for average business cycle statistics than models with frictionless durable adjustment, our model with fixed costs also provides a better fit for other dynamic patterns of consumption as well as for household level microdata.

There is a long line of literature studying models with durable consumption. Baxter [1996] investigates the implications of durables for business cycles in a two-sector representative agent framework. Mankiw [1982], Bernanke [1985] and Caballero [1990] study the implications of durables for test of the permanent income hypothesis, again in a representative agent framework. Bertola and Caballero [1990], Caballero [1993] and Eberly [1994] investigate stylized heterogeneous agent durables models with fixed cost of adjustment and argue that fixed costs can help explain aggregate dynamics. Leahy and Zeira [2005] and Browning and Crossley [2009] argue that the timing of durable purchases can insulate non-durable purchases from shocks. However, these papers must make strong assumptions to obtain analytical results or to simplify the analysis, and are thus less useful for quantitative analysis. In addition our estimation of time-variation in household durable holdings builds on Eberly [1994] as well as Attanasio [2000] who estimate the distribution of households’ desired vehicle holdings in more stylized (S,s) models. However, we have a more realistic, general equilibrium model, and the restrictions necessary to estimate these earlier papers do not hold in our model. Furthermore, we show that, as in our model, the microdata implies an upward sloping adjustment hazard as a function of a households’ current durable gap in contrast to the single (S,s) threshold imposed in these earlier papers.

9While this insulation margin is active in our model, we find that it is quantitatively small relative to the illiquidity effect.
papers.


Perhaps most related is a recent working paper, Iacoviello and Pavan [2009]. They build a similar incomplete markets model with fixed costs of housing adjustment and aggregate shocks, however they focus on different questions. While our model is infinite horizon, they instead build a life-cycle model, and computational considerations then require an annual rather than quarterly frequency. As such, their model is less suited for examining business cycle dynamics. They instead focus on explaining secular changes in aggregate volatility.

The remainder of the paper proceeds as follows: Section 2 presents data on the behavior of durable and non-durable consumption during the most recent recession. We then provide time-series evidence that durable expenditures display time-varying sensitivity to aggregate shocks from 1960-2010. Section 3 presents our representative agent model, calibration and business cycle results. In Section 4 we describe our heterogeneous agent model and show that adding fixed costs improves the fit in the most recent recession. Section 5 discusses the model’s implications for time-varying impulse responses and estimation using PSID data, Section 6 discusses time-varying credit conditions and relative price movements, and Section 7 concludes.
2 Consumption Dynamics

Figure 1 shows the percentage change in detrended output, consumption of durables and consumption of non-durables, beginning in the fourth quarter of 2007. We define non-durable consumption as BEA "non-durable goods" + "services" - "housing services", and we define durable consumption as "durable goods" + "residential investment". GDP is defined as these consumption expenditures plus private non-residential investment.\textsuperscript{11}

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Decline Relative to Trend in 2007 Recession</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>GDP</td>
</tr>
<tr>
<td></td>
<td>5.8%</td>
</tr>
</tbody>
</table>

Table 1 shows the decline in GDP, durable and non-durable spending over the most recent recession. The decline in durable expenditures in the most recent recession accounted for more almost half of the total decline in GDP, despite the fact that the level of durable spending was only 24% of GDP in 2007q4. These numbers are constructed using an HP Filter 1600 trend. Since our model does not feature growth, we believe detrended numbers are the appropriate model comparison. Nevertheless, we have also constructed similar numbers using linear or no detrending, and results are similar.\textsuperscript{12}

While we believe that understanding the behavior of durable expenditures is very important for this recession, the general patterns hold across previous business cycles. Figure 3 shows the behavior of GDP and disaggregated consumption across time.\textsuperscript{13}

\textsuperscript{10}Hp filtered with smoothing parameter 1600. Other detrending methods produced similar results. Without detrending, non-durable consumption continued to rise for several quarters into the recession. Since our model does not have trend growth, we believe that detrended data is the appropriate model comparison.

\textsuperscript{11}To be consistent with our model, we are excluding government spending and net exports from our calculations, but the results are not dramatically altered by their inclusion.

\textsuperscript{12}The level of the declines is altered substantially using different detrending, but the relative declines, which are the focus of our paper are similar.

\textsuperscript{13}From this graph it may appear that durable expenditures are less important for modern business cycles than for recessions in the mid 80s. However, while the volatility of durable expenditures has decreased, the size of durable expenditures as a fraction of GDP has grown, so that the total variance of GDP explained by durables has risen over time.
Table 2 shows that, overall, both non-durable consumption and durable consumption are very procyclical, but durable expenditures are three times more volatile than GDP while non-durable expenditures are moderately less volatile than GDP. The last column of Table 2 shows that similar results obtain when comparing raw growth rates rather than deviations from trend.

<table>
<thead>
<tr>
<th></th>
<th>Relative</th>
<th>Contemporaneous</th>
<th>Relative</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Standard</td>
<td>Contemporaneous</td>
<td>Std. Dev.</td>
</tr>
<tr>
<td></td>
<td>Deviation</td>
<td>Auto-Correlation</td>
<td>with Output</td>
</tr>
<tr>
<td>Output</td>
<td>0.023</td>
<td>1.00</td>
<td>0.90</td>
</tr>
<tr>
<td>Durable</td>
<td>0.070</td>
<td>3.04</td>
<td>0.90</td>
</tr>
<tr>
<td>Non-Durable</td>
<td>0.013</td>
<td>0.57</td>
<td>0.88</td>
</tr>
</tbody>
</table>
In addition to these standard business cycle statistics, we present evidence that durable expenditures are more responsive to aggregate shocks after long expansions than during recessions. As in Bachmann, Caballero, and Engel [2010], we estimate a two-stage time-series model. In the first stage, we estimate an AR process for durable expenditures, and in the second stage, we regress the absolute value of the residuals from the first stage on the average of lagged durable expenditures. (See Appendix 1 for details). We find clear evidence that aggregate durable expenditures exhibit conditional heteroscedasticity. Figure 4 shows that residual variance rises dramatically with previous durable expenditures. At the peak of the nineties boom (when durable investment rates were very high), the residual variance was 50% larger than at the trough of the 2007 recession (when durable investment rates were very low).\footnote{Bootstrapped 90 percent confidence interval in gray.}

In principle, conditional heteroscedasticity could arise for two reasons: (1) The economy is subject to shocks of constant variance but durable expenditures respond more to these shocks during booms
than during recessions. (2) The size of shocks during booms is greater than the size of shocks during
recessions. While durable expenditures exhibit substantial conditional heteroscedasticity, aggregate
non-durable expenditures do not exhibit similar properties. Since the shocks affecting durable and
non-durable expenditures should be similar, we interpret conditional heteroscedasticity of durable
expenditures as evidence of the first explanation: durable expenditures become more responsive to
aggregate shocks during business cycle booms.

In the following sections, we will argue that conditional heteroscedasticity is implied by our model
with fixed costs of durable adjustment while models with flexible durable adjustment or convex ad-
justment costs do not yield heteroscedasticity. This is because in the presence of fixed adjustment
costs, the response of aggregate durable expenditures to shocks depends on the fraction of households
that choose to adjust at each date. After a long business cycle expansion, households have more
pent-up demand for durables and so there are more households on the margin of adjustment. This
in turn increases the responsiveness of aggregate durable expenditures to shocks.

Since housing adjustment is likely to exhibit larger transaction costs than is the adjustment of
consumer durables, it should exhibit greater conditional heteroscedasticity if such heteroscedasticity
is indeed driven by fixed costs of adjustment. Consistent with the mechanism in our model, we
estimate heteroscedasticity separately for consumer durables and housing investment and find that
there is substantially more heteroscedasticity in residential investment than in consumer durables.
(Again see Appendix 1 for details).

To what extent are business cycle models of consumption consistent with these facts? We begin
with a simple representative agent model and examine its implications for aggregate consumption
dynamics.

3 A Representative Agent Model with Durables

We integrate durable consumption into an otherwise standard RBC model with exogenous labor
supply,\textsuperscript{15} and evaluate its ability to match aggregate dynamics of consumption. The planner problem

\textsuperscript{15}While it is straightforward to add labor supply, it barely alters the conclusions and it makes comparisons with the
incomplete markets model more difficult since computational considerations reduce the feasibility of introducing labor
solves:

\[
\max_{C_t, K_{t+1}, D_t} E \sum_{t=0}^{\infty} \beta^t \left( \frac{[C_t^v D_t^{1-v}]^{1-\theta} - 1}{1 - \theta} \right)
\]

s.t.

\[
\begin{align*}
C_t + D_t + K_{t+1} &= Z_t K_t^\alpha H^{1-\alpha} + (1 - \delta_k) K_t + (1 - \delta_d) D_{t-1} \\
\log Z_t &= \rho \log Z_{t-1} + \xi_t.
\end{align*}
\]

The first order conditions and equilibrium characterization are standard, so we leave the details to Appendix 2. Table 3 contains the benchmark parameters, most of which are standard in quarterly business cycle calibrations.

<table>
<thead>
<tr>
<th>Table 3</th>
<th>Model Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
<td>RBC</td>
</tr>
<tr>
<td>(\beta)</td>
<td>0.99</td>
</tr>
<tr>
<td>(\rho)</td>
<td>0.95</td>
</tr>
<tr>
<td>(\sigma_z)</td>
<td>0.008</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>0.3</td>
</tr>
<tr>
<td>(\delta_k)</td>
<td>0.022</td>
</tr>
<tr>
<td>(\delta_d)</td>
<td>0.022</td>
</tr>
<tr>
<td>(\theta)</td>
<td>2</td>
</tr>
<tr>
<td>(\nu)</td>
<td>0.73</td>
</tr>
<tr>
<td>(h)</td>
<td>0.33</td>
</tr>
<tr>
<td>(\rho_\eta)</td>
<td>N/A</td>
</tr>
<tr>
<td>(\sigma_\eta)</td>
<td>N/A</td>
</tr>
<tr>
<td>(f)</td>
<td>N/A</td>
</tr>
</tbody>
</table>

supply in that model.
The discount rate is picked to generate a quarterly interest rate of 1%, and we assume an intertemporal elasticity of substitution\textsuperscript{16} $\theta$ of 2. The depreciation rate of capital $\delta_k = 0.022$ is set to match the long-run average investment to capital ratio. The average depreciation rate of consumer durables is moderately higher than the depreciation rate of capital while the depreciation rate of residential capital is somewhat lower than that on capital, so in our benchmark results we impose an intermediate value and set $\delta_d = \delta_k$. If anything, this is an over estimate of the actual stock weighted depreciation rate, but durable expenditures are even more volatile in the presence of lower depreciation rates, so we view our choice as a conservative one. The relative weight on non-durable consumption\textsuperscript{17}, $v$, is set to match the average ratio of non-durable consumption to investment in durables\textsuperscript{18} of 4.0. Appendix 2 discusses the sensitivity of our results to these parameter choices as well as to relaxing the Cobb-Douglas specification for the utility function and shows that our results are very robust to a range of reasonable parameter choices.

After solving the model, we then compute business cycle statistics. Table 4 shows that the RBC model generates durable expenditures that are almost an order of magnitude too large, relative to the data. Conversely, non-durable expenditures are not volatile enough. Overall, durable expenditures are 38 times more volatile than non-durable expenditures while they are only 5 times as volatile in the data.

\textsuperscript{16}In general, raising the elasticity of substitution makes both non-durable expenditures and durable expenditures less volatile. Since in our benchmark results we find that non-durable expenditures are not volatile enough while durable expenditures are too volatile, altering $\theta$ can improve the fit along one dimension but reduces the fit along the other.

\textsuperscript{17}In all results, we assume a Cobb-Douglas when weighting durable and non-durable consumption. We have also experimented with allowing for a CES specification, and it did not change our qualitative conclusions.

\textsuperscript{18}The value of 4 may seem somewhat low relative to standard numbers from NIPA, but recall that our measure of non-durable expenditures excludes housing services while our measure of durable expenditures includes residential investment. Increasing the value of this target did not affect any of our qualitative conclusions.
### Table 4

#### Business Cycle Volatility

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>RBC</th>
<th>No Fixed Costs</th>
<th>With Fixed Costs</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Standard</td>
<td>Standard</td>
<td>Standard</td>
<td>Standard</td>
</tr>
<tr>
<td>Deviation</td>
<td>Deviation</td>
<td>Deviation</td>
<td>Deviation</td>
<td>Deviation</td>
</tr>
<tr>
<td>Relative to Y</td>
<td>Relative to Y</td>
<td>Relative to Y</td>
<td>Relative to Y</td>
<td>Relative to Y</td>
</tr>
<tr>
<td>Durable</td>
<td>3.04</td>
<td>13.71</td>
<td>14.22</td>
<td>2.81</td>
</tr>
<tr>
<td>Non-Durable</td>
<td>0.57</td>
<td>0.36</td>
<td>0.40</td>
<td>0.51</td>
</tr>
</tbody>
</table>

While we think the overall business cycle statistics are informative, we are also interested in the ability of our models to match the behavior of consumption during the Great Recession. We believe that any business cycle model that is to be useful for policy analysis should be successful at matching the behavior of consumption during this period of dramatic upheaval.

We simulate the Great Recession by picking a sequence of TFP shocks in the model to match U.S. time-series data\(^\text{19}\) together with an unanticipated 4 percent decline in the capital stock in 2008 quarter 4.\(^\text{20}\) We choose these particular shocks because we believe they map typical descriptions of the recession into the mechanisms of our model. The one time decline in the capital stock corresponds to a decline in household wealth while the decline in TFP that is fed into the model (perhaps driven by financial dislocations that are not modeled), leads to a decline in household earnings. We pick the magnitude of the capital shock to roughly match the declines in capital actually observed in the most recent recession.

After the capital shock, all variables transition as usual under the model equilibrium. The upper panel of Figure 5 shows the response of GDP, non-durable and durable expenditures to the recession shock. Clearly, the durable expenditure response dwarfs the response of GDP and is an order of magnitude larger than that observed empirically in Figure 1. Since the decline in durable expenditure

\(^{19}\)Specifically, the shocks are the residuals estimated from the AR process for durable expenditures estimated in Appendix 1. Results are not particularly sensitive to fitting U.S. data using different shock processes.

\(^{20}\)We have also experimented with shocks to the durable stock, but we believe these map less naturally into the recession. The decline in housing value was largely a decline in the price of housing rather than a decline in the real stock of housing. Furthermore, large declines in the housing stock lead to counterfactual housing booms. In Section 6, we add movements in the relative price of durables and non-durables. In this extension, the relative price and thus value of the durable stock falls during the simulated recession even though there is no real shock to the stock of durables.
is so large, the lower panel plots non-durable expenditures and GDP separately. The peak-to-trough decline in non-durable expenditures is only 43% that of GDP, in contrast to the decline in the data which was 64% as large as GDP.

Why is the decline in durable expenditures so large while the decline in non-durable expenditures is so small? Households get utility from the durable stock and from non-durable expenditures. Given the Cobb-Douglas specification for utility, households want to maintain a constant ratio of $C$ to $D$. Since $D$ is by definition durable, a given decline in $C$ must be matched by a much larger decline in durable expenditures. In general non-durable consumption varies little because households can use both $K$ and $D$ to smooth non-durable consumption. Relative to a business cycle model that does not distinguish between non-durable and durable consumption, non-durable consumption is a much smaller fraction of total wealth and is thus relatively easy to smooth.

While it is possible to match the relative volatilities of non-durable and durable expenditures

\footnote{See Appendix 2 for discussion of the extension to CES.}
by introducing quadratic adjustment costs into the RBC model, we view this as unsatisfactory for several reasons. First, there is substantial evidence of lumpy durable adjustment at the micro level which will not be replicated by convex adjustment costs. There is ample evidence for fixed costs of durable adjustment such as broker’s and titling fees, and we will argue that modeling these costs more explicitly matters for aggregate dynamics in a way that is not captured by quadratic adjustment costs. In particular, we will show shortly that an RBC model with quadratic adjustment costs does not generate the conditional heteroscedasticity that we observe in aggregate time-series data, in contrast to a model with fixed costs of durable adjustment.

4 Incomplete Markets Model

4.1 Flexible Durable Adjustment

In the presence of borrowing constraints, some households will be liquidity constrained and their consumption patterns may be dramatically different from those who are unconstrained. Thus, we are interested in the implications of borrowing constraints for consumption dynamics. The benchmark incomplete markets model assumes that heterogeneous agents face uninsurable idiosyncratic shocks to their labor earnings and borrowing constraints, but is otherwise identical to the representative agent model. Households solve

$$\max_{c_t, d_t, a_t} E \sum \beta^t \left( \frac{(c_t^1)^{\theta} (d_t^1)^{1-\theta}}{1-\theta} - 1 \right)$$

s.t.

$$c_t^i = w_t h_t^i + (1 + r_t) a_{t-1}^i + d_{t-1}^i (1 - \delta_d) - a_t^i$$

$$a_t^i \geq 0; \quad d_t^i \geq 0$$

$$\log \eta_t^i = \rho \log \eta_{t-1}^i + \epsilon_t^i \quad \text{with} \quad \epsilon_t^i \sim N(0, \sigma_z),$$
where \( c_i^t, d_i^t \) and \( a_i^t \) are household \( i \)'s non-durable consumption, durable stock, and assets, respectively. \( \eta_i^t \) represents shocks to idiosyncratic labor earnings while \( w_t \) and \( r_t \) are the aggregate wage and interest rate. A representative firm rents capital and labor and its first order conditions pin down these prices:

\[
\begin{align*}
  w_t &= (1 - \alpha) Z_t K_t^{\alpha} H^{1-\alpha} \\
  r_t &= \alpha Z_t K_t^{-1} H^{1-\alpha} - \delta_k \\
\end{align*}
\]

where as before, aggregate productivity evolves as an AR process

\[
\log Z_t = \rho Z \log Z_{t-1} + \xi_t.
\]

As usual, equilibrium requires that the aggregate resource constraint

\[
C_t + D_t + K_{t+1} = Z_t K_t^{\alpha} H^{1-\alpha} + (1 - \delta_k) K_t + (1 - \delta_d) D_{t-1}
\]

be satisfied, where

\[
\begin{align*}
  K_t &= \int a_{t-1}^i \\
  D_t &= \int d_t^i \\
  C_t &= \int c_t^i \\
  H &= \int h \eta_{t}^i.
\end{align*}
\]

Solving the household problem requires forecasting aggregate prices and thus the aggregate capital stock, which is determined by the continuous distribution of household states, so as usual solving the model requires making computational assumptions. Following Krusell and Smith [1998], we conjecture that after conditioning on aggregate productivity, aggregate capital is a linear function of
Given these assumptions, the infinite horizon problem can be recast recursively as

\[
V(a_{-1}, d_{-1}, \eta; Z, K) = \max_{c,d,a} \left[ \frac{c^\gamma d^{1-\gamma}}{1 - \theta} + \beta E_{t+1} V(a, d, \eta' ; Z', K') \right] \\
s.t. \\
a > 0 \\
c = \omega h \eta + (1 + r)a_{-1} + d_{-1} (1 - \delta_d) - d - a \\
r = Z \alpha K^{\alpha-1} H^{1-\alpha} - \delta_k \\
w = Z (1 - \alpha) K^\alpha L^{-\alpha} \\
K' = \gamma_0 (Z) + \gamma_1 (Z) K \\
\log \eta' = \rho_\eta \log \eta + \varepsilon \\
\log Z' = \rho_Z \log Z + \xi
\]

We solve the model by conjecturing an aggregate law of motion, approximating the value function by linearly interpolating\(^\text{23}\) between continuous grid points, solving the contraction, simulating the household problem and updating the aggregate law of motion until convergence is obtained. In equilibrium, the aggregate law of motion is highly accurate. See Appendix 2 for additional details on the solution method.

Table 3 shows the calibration for the incomplete markets model. Most of the parameter values are the same as in the RBC model.\(^\text{24}\) We again pick \(\beta\) and \(\nu\) to match a quarterly interest rate of 1\%

\(^{22}\)The forecasting rule might also depend on the previous durable stock. An earlier version of this paper found that this added little explanatory power and had substantial computational cost.

\(^{23}\)We have found that linear interpolation gives speed advantages that make it attractive relative to cubic spline or other interpolation methods. While linear interpolation will introduce kinks into the value function, we do not rely on derivative based methods for solving the household problem, so this does not prove particularly problematic.

\(^{24}\)While computational complexity prevents a thorough battery of robustness checks, we have done extensive checks of robustness using the simpler RBC model. We are thus confident that the results in the more complicated models are also likely to be robust to a similarly wide range of parameter values.
and a ratio of non-durable expenditures to durable expenditures of 4. The idiosyncratic productivity process is calibrated to match the persistence and variance of annual labor earnings in PSID data. The implied persistence of idiosyncratic earnings is 0.975 while the standard deviation is 0.1.

Table 4 shows that the incomplete markets model generates substantially higher volatility of non-durable consumption and is thus closer to the data than is the representative agent model. This is because borrowing constrained households are unable to smooth their consumption relative to income and so respond much more strongly to the business cycle. However, while non-durable expenditures are amplified in the presence of borrowing constraints, the same logic also amplifies the volatility of durable expenditures. Thus, the incomplete markets model is even worse at explaining the business cycle behavior of durables than is the RBC model.

In addition to the standard business cycle statistics, we again simulate the Great Recession through the lens of our model and compare the model series to those in the data.\textsuperscript{25} \textsuperscript{26} Figure 6

\textsuperscript{25}Beginning from the ergodic distribution, we simulate the same sequence of aggregate shocks as in the RBC model, together with a 4% decline in capital in 2008q4.

\textsuperscript{26}It's worth noting that there is a potential computational complexity with this procedure. Our solution method
shows the response of GDP, and durable and non-durable expenditures. Unsurprisingly, the fit along the non-durable dimension is somewhat better while the ability of the model to match durable expenditures is worsened.

4.2 Fixed Costs of Durable Adjustment

Can realistic durable adjustment costs help to reconcile the model with the data? We assume that households face the following adjustment costs on their durable stock:

\[ A(d, d_{-1}) = \begin{cases} 
0 & \text{if } d \in ((1 - \delta_d) d_{-1}, d_{-1}) \\
 f (1 - \delta_d) d_{-1} & \text{else} 
\end{cases} \]

This specification implies that households can maintain their durable stock or let part of it depreciate without paying an adjustment cost, but if they want to adjust their durable stock by larger amounts then they must pay a fixed adjustment cost. Thus, we allow households to engage in routine maintenance that does not incur a fixed cost but assume that they must pay a cost when actually buying or selling their current durable stock. We associate these fixed costs with explicit transaction costs together with time costs.

This adjustment cost specification lies between two extremes that are common in the literature. Under one extreme, households must pay the adjustment cost if \( d \neq d_{-1} \). This specification implies that households cannot let their houses depreciate without paying a fixed cost. Alternatively, it is also common to assume that the adjustment cost occurs if \( d \neq (1 - \delta_d) d_{-1} \). Under this specification, households cannot maintain their durable stock without paying an adjustment cost. While these alternatives are somewhat simpler, we think that our specification is likely to better capture the realities of the costs associated with durable adjustment. Nevertheless, we have experimented with these alternative specifications and it did not materially affect our results.

requires using Krusell-Smith methods to approximate the distribution of household states. While this approximation is highly accurate throughout the ergodic distribution, it is possible that approximations may be far from accurate following the one-time recession shock. However, we find that forecasts are only mildly worsened. See Appendix 2 for further discussion.

27 Brokers fees, titleing fees, etc.

28 In these extreme cases, when households choose to not buy or sell durables, the consumption decision becomes one-dimensional. In our specification, households must still choose how much to let the durable stock depreciate.
With this adjustment cost specification, the aggregate resource constraint must now include adjustment costs

\[ C_t + D_t + K_{t+1} + A_t = Z_t K_t^\alpha H^{1-\alpha} + (1 - \delta_h) K_t + (1 - \delta_d) D_{t-1}, \]

where

\[ A_t = \int A(d^i, d^p). \]

The recursive formulation of the problem can now be written as

\[
V(a_{-1}, d_{-1}, \eta; Z, K) = \max \left[ V^{\text{adjust}}(a_{-1}, d_{-1}, \eta; Z, K), V^{\text{noadjust}} \right]
\]

with

\[
V^{\text{adjust}}(a_{-1}, d_{-1}, \eta; Z, K) = \max_{c,d,a} \frac{[c^\delta d^{1-\delta}]^{1-\theta}}{1-\theta} + \beta E_{\xi} V(a, d, \eta'; Z', K')
\]

s.t.

\[
c = \text{wh} \eta + (1 + r)a_{-1} + d_{-1} (1 - \delta_d) - d - a - f (1 - \delta_d) d_{-1}
\]

\[
a > 0; \quad \text{equilibrium conditions and prod. processes}
\]

\[
V^{\text{noadjust}}(a_{-1}, d_{-1}, \eta; Z, K) = \max_{c,d,a} \frac{[c^\delta d^{1-\delta}]^{1-\theta}}{1-\theta} + \beta E_{\xi} V(a, d, \eta'; Z', K')
\]

s.t.

\[
c = \text{wh} \eta + (1 + r)a_{-1} + d_{-1} (1 - \delta_d) - d - a
\]

\[
a > 0; \quad \text{equilibrium conditions and prod. processes}
\]

We now approximate the value function of adjusting and not adjusting separately and then solve as before. Again, see Appendix 2 for additional details. We again calibrate \( \beta \) and \( \upsilon \) to match an interest rate of 1% and a non-durable to durable expenditure ratio of 4. We set the fixed cost of adjustment at 2.5%, so that households lose 2.5% of their pre-adjustment durable stock when adjusting outside of the range of depreciation.\(^{29}\)

\(^{29}\)This is one half of the value used by Diaz and Luengo-prado [2010], who report that the typical fee charged by real estate brokers in the U.S. economy is around 6%. Since the fixed cost of consumer durable adjustment are smaller, we pick a lower value for our benchmark. In our model, this implies a quarterly adjustment frequency of just under 3%
After solving the model, we again compute business cycle statistics, which are displayed in Table 4. The model with fixed costs generates non-durable expenditures which are more volatile and durable expenditures which are less volatile than the previous models. It is thus a dramatic improvement over the models without adjustment costs.

Figure 7 shows that, in contrast to the models flexible durable adjustment, the model now does a good job of reproducing the recession of 2007. GDP now declines by 6.5%, non-durable expenditures decline by 4% and durable expenditures decline by 24%. The relative decline in durable expenditures is still slightly too large, but the fit is remarkably close given that none of the declines are specifically targeted in the calibration. This is in marked contrast to the models with flexible durable adjustment that imply declines in durable expenditures that are an order of magnitude too large.

Clearly, the presence of adjustment costs dampens the response of durable expenditures to aggregate and that adjustment occurs on average every 8 years. While this is higher than the frequency of adjustment of housing it is somewhat lower than the frequency of consumer durable adjustment in the data. Our general conclusions were not sensitive to decreasing adjustment costs to 2% or raising adjustment costs to 10% as long as \( \nu \) is recalibrated to maintain a constant ratio of non-durable to durable expenditures.

23
gate shocks. In addition, the model now matches the decline in non-durable expenditures. This is because the presence of adjustment costs on durables means that a fraction of household wealth is effectively illiquid. Since we target the same level of total wealth in all models, a greater fraction of households are temporarily liquidity constrained in the model with fixed costs and thus illiquid wealth. These households are similar to the wealthy hand-to-mouth households emphasized in Kaplan and Violante [2011]. While households may have a large amount of total wealth, households rationally choose to avoid paying fixed costs associated with using illiquid wealth to smooth non-durable consumption. In their model, wealth is illiquid because households lock some wealth into a higher return investment while in our model, wealth is illiquid because households want to consume durables, which are subject to adjustment costs. While the general mechanism is similar, we have additional panel data on income, durable and non-durable expenditures and wealth, which allows us to test directly for this mechanism.\[^{30}\]

5 Non-Linear Dynamics

In section 2, we showed that aggregate durable expenditures exhibit conditional heteroscedasticity: durable expenditures are more volatile towards the end of expansions than during recessions. To what extent are our models consistent with this time-series evidence? As in Bachmann, Caballero, and Engel [2010], Appendix 1 presents estimates of durable expenditure conditional heteroscedasticity for the model with fixed costs of durable adjustment as well as the models with no cost of durable adjustment and for an RBC model with quadratic adjustment costs. The model with fixed costs of durable adjustment implies conditional heteroscedasticity that is in line with the empirical estimates while the model with flexible durable adjustment exhibits no such conditional heteroscedasticity.

Why does the model with fixed costs of adjustment imply heteroscedasticity despite the fact that the shocks hitting the model are of constant variance? Fixed costs of adjustment imply that the individual households’ response to aggregate shocks is highly non-linear. Since households are

\[^{30}\]Previous versions of this paper explored the implications of this illiquid wealth for household level insurance using Italian Survey of Household Income and Wealth Data. In ongoing work, we explore these implications using PSID data.
continuously hit with both aggregate and idiosyncratic shocks, in the presence of fixed costs of durable adjustment there is a gap between their current durable holdings and the durable holdings they would choose if they temporarily faced no adjustment costs. When this gap is small, it is not worth paying the fixed cost of adjustment. As this gap becomes large, it becomes optimal to pay the fixed cost. Thus, fixed costs of adjustment induce lumpy durable purchases, and changes across time in the fraction of households choosing to adjust can induce significant time-variation in aggregate durable expenditures. Mechanically, time $t$ aggregate durable expenditures are given by $X_t = \int (d_{i,t} - d^*_{i,t}) h_t ((d_{i,t} - d^*_{i,t}))$ where the "durable gap" $(d_{i,t} - d^*_{i,t})$ is the difference between household $i$’s current durable stock (after maintenance), $d_{i,t}$, and the stock it would choose if it adjusts, $d^*_{i,t}$, and $h_t ((d_{i,t} - d^*_{i,t}))$ is the adjustment hazard as a function of the durable gap.\footnote{Where the gaps and hazards are defined relative to durable holdings after maintenance so that $X_t$ is durable investment excluding maintenance.} If more households choose to adjust in response to an aggregate shock, then aggregate durable expenditures will respond more strongly to the shock.

Figure 8 plots the distribution of durable gaps and adjustment hazard in a boom and in a recession,
for the model with fixed costs of durable adjustment. The distribution of durable gaps is skewed right because depreciation means that more households want to increase than to decrease durable holdings. During the boom, households' desired durable holdings rise so that the distribution of durable gaps shifts to the right. As more households are now further from their desired level of durables, they move into the region with a higher probability of adjustment. Since all households that adjust will respond to aggregate shocks, aggregate durable expenditures become more responsive to these shocks. This is amplified by the increase in the probability of adjustment during a boom. Households are more likely to adjust up and less likely to adjust down for a given durable gap (as can be seen by the shifting hazard in Figure 8), and the net effect is an increase in the probability of adjustment. (It is worth noting that aggregate durable expenditures become more responsive to both positive and negative aggregate shocks during a boom. During a boom, more households are in the region with a steep hazard of adjustment, and these households become more responsive to both positive and negative aggregate shocks).
Figure 9 shows the impulse response to a one standard deviation increase in TFP in 1999 and compares it to the impulse response calculated in 2009.\textsuperscript{32} As suggested by the above discussion, the impulse response is much larger during a boom than during a recession. The cumulative impulse response in 1999 is roughly twice that in 2009. Appendix 1 provides additional evidence of procyclical impulse response functions and compares model estimates to time-series evidence.

Thus our model suggests that policies designed to simulate durable expenditures during recessions may have less effect than suggested by linear VAR evidence that assumes there is no time-variation in the response to shocks. For example, both "Cash for Clunkers" and "First-Time Home Buyers Tax Credit" were enacted to stimulate durable expenditures during the most recent recession. However, if pent up durable demand was relatively low, then this may have dampened the effectiveness of the programs. Conversely, if each program led to an increase in durable demand and pushed more households towards an active region of adjustment, then the total effect of the two programs may have been greater than the sum of the individual programs in isolation. We find support for such non-linearities by computing the impulse response to TFP shocks of varying sizes. We find that doubling the size of the TFP shock roughly triples the cumulative impulse response to the shock.

Given the import policy implications of these non-linearities, we now look for direct evidence that the cross-sectional distribution of household durable demand moves in the manner predicted by our model. The obvious difficulty with constructing an empirical counterpart to Figure 8 is that "durable gaps" are not observed in the microdata. However, our structural model imposes strong restrictions on the relationship between variables which are observed in microdata and durable gaps, which are observed in the model but not in the data.\textsuperscript{33} We use the model to estimate a flexible functional form that relates \(d_{t-1}, c, a\) to the durable gap, and find that within the model these variables are sufficient.

\textsuperscript{32}The hump-shaped IRFs eventually return to zero. Interestingly the hump-shape is consistent with the impulse response of aggregate durable expenditures to observed changes in TFP in the data. The hump-shape arises due to equilibrium movements in the interest rate across time. On impact, as TFP increases, interest rates rise, which increases financial income as well as the return to saving in liquid assets. So initially, most of the increase in savings goes into liquid assets. However, as additional capital is accumulated, the return to saving in liquid assets falls and households begin to accumulate more in durable assets so that the response of durable expenditures grows with time. As the TFP shock dies out, this process reverses itself and the economy returns to steady-state.

\textsuperscript{33}Our empirical strategy in general is similar to that in Eberly [1994], but we have a more complicated empirical model that does not require us to exclude borrowing constrained households. Furthermore, we test the predictive power of our estimated durable gaps for the actual probabilities of adjustment, and find an upward sloping adjustment hazard as predicted by our model. In contrast, Eberly [1994] imposes and estimates a single adjustment threshold.
to explain more than 99% of the variation in durable gaps. Once we have estimated the relationship in the model, we then apply the same relationship to measures of $d_{-1}, c, a$ in PSID microdata, which allows us to estimate an empirical cross-section of durable gaps for each of the sample years in the PSID from 1999-2009. Finally, once we estimate the distribution of durable gaps in the data, we then also compute the empirical adjustment hazard as a function of the estimated durable gaps. This provides an extremely strong test of whether our estimated durable gaps have predictive power for actual adjustment behavior and whether they shift over the business cycle in a manner consistent with our model. See Appendix 3 for additional details on the estimation procedure and sample construction.

Figure 2 compares the estimates from the PSID data to the predictions from our model. Given that our model is abstracting from life-cycle considerations and many other idiosyncratic taste shocks that will affect durable adjustment, the fit between the model and the data is remarkably strong. Our estimated durable gaps have strong predictive power for the probability of adjustment. The empirical probability of adjustment rises from approximately 20% for a durable gap of zero to more than 50% for a durable gap of 0.5. (While the adjustment probability of 20% at zero may appear to be a failure, it is worth noting that our measure of adjustment is calculated over a two-year period while our gap measure is instantaneous, so a positive probability despite a gap of zero may reflect a positive gap at some other point during the two year period which led to adjustment). In addition to these strong predictive power on average, we also find that both the empirical distribution of durable gaps as well as the corresponding adjustment hazards shift between 1999 and 2009 in a way that is remarkably consistent with our model. Thus, there is strong empirical evidence that both the distribution of households’ durable gaps and adjustment probabilities move in the manner necessary to generate a procyclical impulse response.\footnote{While Figure 9 shows that the IRF on impact is only a partial summary of the differences in the cumulative IRFs, we can nevertheless compute the IRF on impact implied by our empirical distribution of gaps and hazards (ignoring GE effects) and we find that the implied IRF on impact is approximately 20 percent larger in 1999 than in 2009.}
6 Model Extensions and Robustness

So far our model has abstracted from two features of durables that may be particularly important in the most recent recession: tightening credit conditions and movements in the relative price of durables. In this section we briefly investigate extensions of the model along these dimensions and argue that they do not have large impacts on our conclusions.

Thus far we have assumed that there is no borrowing in the economy and that durables must be financed wholly with current spending. Empirically, many durables are purchased with down payments with the durable itself used as collateral for the loan. In addition, there was a decline in the relative price of durables during the most recent recession. Since our utility specification implies that in the frictionless model, households have constant expenditure shares in durables and non-durables, relative price movements are a potentially powerful force. (However, it is worth noting that while the price of durables declined in the Great Recession, over the entire period from 1960-present, the NIPA deflator for durables is actually countercyclical, so relative price movements are unlikely to explain overall business cycle volatilities). Furthermore, there is reason to believe that these two extensions may have important interactions: as the relative price of durables falls, the value of collateral falls, which tightens borrowing constraints.

Endogenizing relative price movements is beyond the scope of this paper as it induces additional computational burden into an already challenging model. However, it is straightforward to introduce exogenous movements in the relative price of durables through an exogenous shock to the rate of transformation between durables and non-durables that is perfectly correlated with aggregate productivity. That is, we assume that the aggregate resource constraint is now given by

\[ C_t + P(Z_t)D_t + K_{t+1} + A_t = Z_t K_t^\alpha H^{1-\alpha} + (1 - \delta_k) K_t + (1 - \delta_d) D_{t-1} \]

where \( P(Z_t) \) is increasing with \( Z_t \) so that the relative price of durables is procyclical.\(^{35}\) In addition to this exogenous price change, we also introduce a collateralized borrowing constraint as in Diaz and Luengo-prado [2010]. Instead of \( a \geq 0 \) we assume that \( a \geq - (1 - \kappa) P(Z_t) D \) where \( \kappa \) is the required collateral.

\(^{35}\)Household budget constraints are modified accordingly.
downpayment. This specification implies that there is an interaction between the price of durables and borrowing constraints. As the price of durables falls, there is a direct price effect that means households want to hold more durables. However, the decline in the price of durables also means that the collateral value of durables falls so that holding durables becomes less attractive\textsuperscript{36}. These effects work in opposite directions, so we again need the quantitative model.

How do these extensions affect consumption dynamics in the model? Before simulating the model, we must fix values for the new coefficients. We set $\kappa = 0.33$. This value is somewhat above typical downpayments in the housing market and reflects the fact that it is more difficult to get loans against other consumer durables. We set the standard deviation of relative prices to be 1.5 times that of TFP so that the relative price of durables falls by 1.5\% in the recession simulation. This may initially seem like a small relative price decline relative to many discussions of the most recent recession, but it reflects several facts. First, residential investment is only approximately one-third of total durable expenditures, and consumer durable prices fell less in the most recent recession than did the price of housing. Second, NIPA deflators for residential investment as well as consumer durable expenditures are less volatile than the Case-Shiller Index and declined less in the recession. Finally, both non-durable and durable prices fell in the recession, so the change in the relative price is less than the individual decline.

Figure 10 shows the simulated recession for the model with and without adjustment costs. Again, the model with fixed costs is markedly more successful. The model with fixed costs now generates a decline in durable expenditures that is moderately too large, but the decline is well within reasonable ranges, in contrast to the huge decline generated by the model with no adjustment costs. In addition, non-durables are now somewhat too volatile in the model with fixed costs of durable adjustment, but both volatilities can easily be improved with mild modifications of the benchmark parameters. In addition, the introduction of collateral constraints and relative price movements does not affect our conclusions about the models’ ability to match the heteroscedasticity observed in time-series data.

Thus, this preliminary evidence suggests that relative price movements and time-varying borrowing constraints may have some mild quantitative implications for our results, but they are unlikely to

\textsuperscript{36}Guerrieri and Lorenzoni [2011] similarly investigate the effects of an exogenous tightening in credit constraints.
overturn any of our qualitative conclusions.

7 Conclusion

Durable expenditures are an important part of the business cycle, yet typical business cycle models abstract from durables entirely. Such models, by assumption, fail to account for the fact that a very large fraction of the most recent recession was explained by declines in durable expenditures. In this paper, we argue that simple extensions of these models that assume frictionless durable adjustment are no more successful at matching business cycle behavior. Business cycle models with frictionless durable adjustment imply a decline in durable expenditures in the most recent recession an order of magnitude larger than what was actually observed. Furthermore, these models feature significant insulation of non-durable consumption so that the implied decline in non-durable expenditures is too low relative to that observed in the recession.
It is widely recognized that there are substantial fixed costs of durable adjustment. Nevertheless, business cycle models with fixed costs of durable adjustment have received limited attention due to their computational complexity. In this paper, we argue that while the introduction of fixed costs of adjustment is computationally challenging, it is not intractable, and it provides a significantly better fit for both macro and micro dynamics. An incomplete markets model with fixed costs and aggregate shocks is able to match the decline in durable expenditures in the Great Recession as well as business cycle volatilities more generally. In addition, since durable holdings are also a form of household wealth, fixed costs of adjustment make a fraction of wealth illiquid and costly to adjust. Since some households have large durable holdings with small levels of liquid wealth, this amplifies the response of non-durable consumption to shocks and implies that the model is able to match the relatively large decline in non-durable expenditures during the recession.

More importantly, introducing realistic adjustment costs is important for evaluating the efficacy of fiscal stimulus. Fixed costs of durable adjustment induce significant non-linear impulse responses of aggregate durable expenditures to shocks. Our estimates imply that during the Great Recession, durable expenditures were half as responsive to additional stimulus as during the late 90s. This time-varying IRF induces conditional heteroscedasticity in the time-series behavior of aggregate durable expenditures: the residuals from an AR regression on durable expenditures are greater during booms than during recessions. We show that such conditional heteroscedasticity is present in actual U.S. time-series data.

The procyclical IRF in the model is driven by changes in the distribution of households’ desired durable holdings over the cycle. During a boom, households have more pent-up durable demand and are more likely to adjust their durable holdings. We test this implications of our model by estimating the empirical distribution of household level desired during holdings using PSID microdata, and we find that the empirical distribution moves as predicted by our model. Thus, the microdata is consistent with a large time-varying IRF, as implied by our model.
References


8 Appendix 1: Empirical Results

8.1 Data Adjustments and Construction

We define durable expenditures as real consumer durable expenditures + real residential investment where real consumer durables are NIPA Table 1.1.5 line 4 divided by NIPA Table 1.1.9 line 4 and real residential investment is NIPA table 1.1.5 line 12 divided by NIPA Table 1.1.9 line 12. Non-durable consumption is defined as non-durable goods (NIPA Table 1.1.5 line 5 divided by NIPA Table 1.1.9 line 5) + services (Table 1.1.5 line 6 divided by Table 1.1.9 line 6) - housing services (Table 2.3.5 line 14 divided by Table 2.4.4 line 14). Our measure of GDP is then the sum of non-durable consumption, durable expenditures and private non-residential investment.

We have also experimented with using analogous constructions with chained GDP rather than constructing real GDP with the GDP deflator. However, the necessary chained GDP components are only available going back to 1995. Nevertheless, the results for the most recent recession are similar. In addition, it should be noted that due to the construction of the price deflators, the real series constructed by deflating the nominal series individually do not add up to the aggregate real series. This problem is more problematic the further from the base year we move. Since the current NIPA data uses 2005 as the base year, this is likely to introduce only small errors into our results for the most recent recession, but it makes the older series somewhat less reliable. The business cycle volatilities we estimate are sensitive to the exact procedure used to construct the individual consumption components (e.g. using non-durable consumption + services - housing services to get our measure of non-durable consumption does not obtain the exact same series as using PCE - housing services - durable expenditures even though these two measures contain the same components). These series are nearly identical in the 2000s, but diverge as we move further back in time.

Constructing durable expenditure rates requires constructing quarterly measures of the durable stock. Following Bachmann, Caballero, and Engel [2010], we construct measures of real quarterly durable expenditures using nominal data from BEA Domestic Product and Income Tables 1.1.5 and price deflators from Table 1.1.9. We then next construct quarterly depreciation estimates using annual nominal measures of depreciation from BEA Fixed Asset Table 1.1 together with the price
deflators from Table 1.1.9. Since the BEA publishes annual measures of the stock of durables and housing in Fixed Asset Table 1.1, we just need to construct quarterly measures in between these annual observations. To do this, we combine the annual observations with the quarterly expenditure and depreciation measures together with a standard stock accumulation expression to construct quarterly stock measures. See Bachmann, Caballero, and Engel [2010] for the more detailed procedure.

8.2 Conditional Heteroscedasticity

In this section we show that durable expenditures exhibit conditional heteroscedasticity, rising in booms and falling in recessions. As in Bachmann, Caballero, and Engel [2010], we assume that our series of interest can be described by an AR process:

\[ x_t = \sum_{j=1}^{p} \phi_j x_{t-j} + \sigma_t \varepsilon_t, \]

where \( x_t = \frac{D_t}{D} \) is durable expenditures divided by the durable stock,\(^{37} \varepsilon_t \sim \text{i.i.d. with zero mean and unit variance}, and

\[
\begin{align*}
\sigma_t &= \alpha + \eta x_{t-1} \\
x_{t-1} &= \frac{1}{k} \sum_{j=1}^{k} x_{t-j}.
\end{align*}
\]

That is, we allow the variance of the residuals in the AR process for durable expenditures to vary with past durable expenditures. This specification implies that the impulse response of \( x \) to \( \varepsilon \) on impact at time \( t \) is given by \( \alpha + \eta x_{t-1} \). If \( \eta = 0 \) then the impulse response of \( x \) to \( \varepsilon \) does not vary with past durable expenditures while \( \eta > 0 \) implies that the IRF rises with lagged expenditures.

We estimate the time-series model using quarterly data on \( \frac{D_t}{D} \) from 1960-2010. The estimation follows a 2-stage procedure. In the first stage, we estimate the AR process via OLS to obtain residuals

\(^{37}\text{The ratio of durable expenditures to the stock is stationary while durable expenditures are not.}\)
The second stage then estimates by OLS $\eta_1$ using

$$|\varepsilon_t| = \left(\frac{2}{\pi^2}\right)^{1/2} (\alpha + \eta \bar{x}_{t-1}) + \text{error}.$$ 

We repeat the estimation for all combinations of $p, k \leq 12$ and choose the best fit, $p^*, k^*$ using AIC. For more details on the time-series model, see Bachmann, Caballero, and Engel [2010]\textsuperscript{38}. Table 6 contains the time-series estimates. Both total durable expenditures as well as residential investment exhibit strongly significant\textsuperscript{39} $\eta > 0$. The estimated $\eta > 0$ for consumer durables, but it is only marginally significant. While $\eta > 0$ implies that there is a statistically significant increase in the IRF with lagged expenditures, it does not imply that the increase is economically significant. In the 4th and 5th rows of Table 6, we report statistics that show that there is quantitatively large variation in the IRF across time. The maximum IRF is 2.82 times larger than the minimum IRF while the 95th percentile is 1.82 times higher than the 5th percentile. Thus, the estimated heteroscedasticity is both statistically and economically significant.

Table 6

<table>
<thead>
<tr>
<th>Conditional Heteroscedasticity Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta$ :</td>
</tr>
<tr>
<td>$t$-$\eta$ :</td>
</tr>
<tr>
<td>Bootstrap p-value ($\eta &gt; 0$) :</td>
</tr>
<tr>
<td>$\pm (\sigma_{\text{max}}/\sigma_{\text{min}})$ :</td>
</tr>
<tr>
<td>$\pm (\sigma_{95}/\sigma_5)$ :</td>
</tr>
<tr>
<td>no. obs. :</td>
</tr>
</tbody>
</table>

\textsuperscript{38}In addition to the model we presented, their paper presents an alternative time-series model. We obtained similar results for this model, so for brevity we did not report these results.

\textsuperscript{39}The bootstrap p-value row constructs bootstrapped p-values for $\eta > 0$, accounting for the fact that errors in the first stage estimation increase the standard errors in the second stage.
Table 7

Conditional Heteroscedasticity Models

<table>
<thead>
<tr>
<th>Series:</th>
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<th>RBC Adj Costs</th>
<th>No Fixed Costs</th>
<th>W/ Fixed Costs</th>
</tr>
</thead>
<tbody>
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<td>( \eta )</td>
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<td>-0.003</td>
<td>0.02</td>
<td>0.08</td>
</tr>
<tr>
<td>( t-\eta )</td>
<td>-0.34</td>
<td>-0.14</td>
<td>0.42</td>
<td>2.13</td>
</tr>
<tr>
<td>( \pm (\sigma_{\text{max}}/\sigma_{\text{min}}) )</td>
<td>1.25</td>
<td>1.24</td>
<td>1.17</td>
<td>1.83</td>
</tr>
<tr>
<td>( \pm (\sigma_{95}/\sigma_5) )</td>
<td>1.15</td>
<td>1.14</td>
<td>1.11</td>
<td>1.49</td>
</tr>
</tbody>
</table>

It is important to note that while we have interpreted conditional heteroscedasticity as a time-varying impulse response to aggregate shocks with a constant variance, an alternative interpretation is that aggregate shocks themselves are larger during booms than during recessions. We test for this by performing the same regressions on Baxter-King bandpass filtered non-durable consumption.\textsuperscript{40} Table 6 shows that, there is no evidence of conditional heteroscedasticity for non-durable expenditures. Presumably the aggregate shocks hitting \( C \) should be similar to the aggregate shocks hitting \( D \), so we interpret our results as evidence that it is not the shocks that drive heteroscedasticity, and is rather the mechanism that translates those shocks into durable expenditures that drives our estimates. Bachmann, Caballero, and Engel [2010] provide additional evidence that aggregate GDP also does not exhibit conditional heteroscedasticity.

To construct Figure 4, we allowed for a more flexible non-parametric second stage. We also bootstrapped the procedure to construct corresponding confidence intervals. To do this, we redraw residuals from the first stage to create one-thousand bootstrap samples when estimating the non-parametric second-stage. We tried a variety of bandwidths for the second-stage kernel estimator, and it did not change the qualitative conclusions.

In addition to these empirical estimates, we also compute heteroscedasticity estimates for our simulated models. Since we know that the true shocks in the model follow an AR(1) process, we report benchmark results restricted \( k = 1, p = 1 \), but results are not sensitive to this restriction. Table 7 shows that the frictionless model does not generate procyclical IRFs. If anything, the

\textsuperscript{40}Unlike expenditure rates, non-durable consumption is non-stationary and so must be filtered. Using alternative filters did not substantively change the results.
model without fixed costs implies $\eta < 0$. In contrast, the model with fixed costs exhibits conditional heteroscedasticity that is in line with the empirical estimates. The estimated $\eta > 0$, and the time-variation in the impulse response on impact is similar to that in the data.

Tables 6 and 7 summarize statistical evidence for a time-varying impulse in the data as well as in the model with fixed costs of durable adjustment, but it is also useful to plot our resulting time-series estimates. Figure 11 shows the estimate of $\sigma$ (which again is an estimate of the IRF on impact) at each date for the time-series model and compares it estimates from the heterogeneous agent models with and without fixed costs of durable adjustment. To construct the estimates for the heterogeneous agent models, we first estimate the time-series model above and then use the residuals from this model as the shock process that we feed into our structural models, beginning from the ergodic distribution. We then reestimate the time-series model on our simulated data and compute the resulting IRFs.

The impulse response for the model with fixed costs closely matches the empirical estimates, rising during booms and falling during recessions. In contrast, there is little time-variation for the model
with no fixed costs.

While fixed costs induce procyclical impulse response functions, they need not be the only mechanism that can generate these dynamics. Another possible explanation for a procyclical IRF is the presence of collateral constraints. If collateral constraints tighten during recessions, then durable expenditures may respond less to shocks during recessions than during booms. We have tested for this using the models with collateral constraints in Section 6. However, we find that this mechanism is quantitatively weak. This is partially because the response to time-varying credit conditions is solely one-sided. A tightening of credit conditions has a direct effect on households that want to increase durables but has no direct effect on households that want to sell durables. This substantially dampens the scope for time-varying IRFs. Furthermore, we find no empirical support for such asymmetric heteroscedasticity.

9 Appendix 2: Model Solutions

9.1 RBC Model

The RBC model (for the Cobb-Douglas case) has the following first order conditions:

\[ C_t : vC_t^{\alpha-1}D_t^{1-v}[C_t^{\alpha}D_t^{1-v}]^{-\theta} = \lambda_t \]
\[ K_{t+1} : \lambda_t = \beta E \left[ \lambda_{t+1} \left( \frac{Y_{t+1}}{K_{t+1}} + (1 - \delta_k) \right) \right] \]
\[ D_t : \lambda_t = (1 - v)D_t^{-v}C_t^{v}[C_t^{v}d_t^{1-v}]^{-\theta} + \beta E \lambda_{t+1}(1 - \delta_d) \]
The steady-state equations (fixing $Z = 1$) are then given by

\[ Y = K^\alpha \]
\[ \lambda = vC^{\alpha-1}D^{1-v} \left[ C^\alpha D^{1-v} \right]^{-\theta} \]
\[ 1 = \beta \left[ \left( \frac{\alpha Y}{K} + (1 - \delta_k) \right) \right] \]
\[ \lambda = (1 - \nu)D^{-\nu}C^{\alpha} \left[ C^\alpha D^{1-v} \right]^{-\theta} + \beta \lambda (1 - \delta_d) \]
\[ C = ZK^\alpha - \delta_k K - \delta_d D \]

Solving for the steady-state gives:

\[ \frac{C}{D} = \frac{v(1 - \beta(1 - \delta_d))}{1 - \nu} \tag{1} \]
\[ K = \left[ \frac{1}{\alpha} \left[ \frac{1}{\beta} - (1 - \delta_k) \right] \right]^{\frac{1}{\nu - 1}} \tag{2} \]
\[ Y = K^\alpha \tag{3} \]
\[ D = \frac{Y - \delta_k K}{\frac{v(1 - \beta(1 - \delta_d))}{1 - \nu} + \delta_d} \tag{4} \]

We set $\delta_k = .022$ to match the long-run ratio of investment to capital, and we set $\alpha = .3$ to match the long-run labor share. We then pick $\beta$ so that the capital stock in 2 implies an interest rate (in a decentralized economy) of 1%. We then pick $\nu$ so that 1 implies $\frac{C}{\delta_d D} = 4$. Given the steady-state solution, we then compute the dynamic solution to the model by taking a second order perturbation around the steady-state solution.\textsuperscript{41}

To investigate the sensitivity of our conclusions to our parameter values and calibration targets, we have also resolved the model using a variety of different specifications. We varied $\delta_d$, choosing 12 values evenly spaced between .011 and .16, 7 values for $\theta$ evenly spaced between 0.5 and 3, three values for the $\frac{C}{\delta_d D}$ (4, 5 and 6), and we also allowed for a CES utility specification with elasticities ranging from 0.7 to 1.7.\textsuperscript{42} Figure 12 shows the resulting standard deviations of durable and non-

\textsuperscript{41} Additional details available on request.
\textsuperscript{42}This includes the range typically estimated in the literature. (See Krueger and Fernandez-Villaverde [2010]). Expanding this range further did not matter for the general conclusions. For high enough elasticity, we can match the volatility of non-durable expenditures relative to output, but for these parameter values, durable expenditures are more
Figure 12: Robustness of RBC results

durable expenditures relative to GDP. None of the parameter specifications (including allowing for CES between durable and non-durable consumption) is able to match the empirical volatility of non-durable expenditures, and only a tiny subset of parameters delivers durable expenditures that are not substantially more volatile than the data.

than 40 times too volatile.
9.2 Incomplete Markets Model

We describe the solution for the model with fixed costs. The solution for the model without fixed costs is similar. Reproducing the recursive formulation from the text, households solve

\[
V(a_{-1}, d_{-1}, \eta; Z, K) = \max \left[ V^{\text{adjust}}(a_{-1}, d_{-1}, \eta; Z, K), V^{\text{noadjust}} \right]
\]

with

\[
V^{\text{adjust}}(a_{-1}, d_{-1}, \eta; Z, K) = \max_{c,d,a} \left[ \frac{c^\alpha d^{1-\nu}}{1-\theta} + \beta E_{\xi} V(a, d, \eta'; Z', K') \right]
\]

subject to

\[
c = w\eta + (1 + r)a_{-1} + d_{-1}(1 - \delta_d) - d - a - f(1 - \delta_d)d_{-1}
\]

equilibrium conditions and production process

\[
V^{\text{noadjust}}(a_{-1}, d_{-1}, \eta; Z, K) = \max_{c,d} \left[ \frac{c^\alpha d^{1-\nu}}{1-\theta} + \beta E_{\xi} V(a, d, \eta'; Z', K') \right]
\]

subject to

\[
c = w\eta + (1 + r)a_{-1} + d_{-1}(1 - \delta_d) - d - a
\]

equilibrium conditions and production process

We begin by substituting the budget constraint into the utility function to eliminate non-durable consumption as a choice-variable. We discretize \( \eta \) and \( Z \) using the algorithm of Tauchen [1986] and approximate \( V^{\text{adjust}}(\cdot; \cdot; \eta; Z;) \) and \( V^{\text{noadjust}}(\cdot; \cdot; \eta; Z;) \) as multilinear functions in the two continuous idiosyncratic states and one continuous aggregate state. Initializing the grid for aggregate capital requires knowledge of the steady-state level of capital, so before solving the model with aggregate shocks, we solve for the steady-state of the model. The solution method is similar and simpler than the solution with aggregate shocks, so we only describe the latter:

Given an initial guess for the value functions and transition function, we solve for the optimal two-dimensional policy functions using a Nelder-Meade algorithm initialized from 3 different starting values to reduce the problems of finding local maxima in the policy function. The values of adjusting

\footnote{We have experimented with cubic spline interpolation and have found that the speed advantages of linear interpolation appear to be worth potential decreases in accuracy (especially since fixed costs imply that the value functions may not be well approximated by cubic splines).}
and not adjusting are compared, to generate the overall policy function and to update the overall value function. We iterate until the separate value functions change by less than 0.001. Once the value functions have converged, we then solve for the optimal policy function an additional time on a finer grid, to use for simulation.

We then simulate a panel of households and compute the evolution of the aggregate capital stock to update the aggregate transition rule $K' = \gamma_0(Z) + \gamma_1(Z) K$. We then repeat the above procedure until the coefficients in the value function change by less than 1%. Once the transition rule has converged, aggregate forecasts are highly accurate, with $R^2 > 0.999$. We have experimented with including the aggregate durable stock in the transition rule and found that it did little to improve forecasts, at considerable additional computational cost.

For the benchmark results, we use 25 grid points each for interpolating $a_{-1}$ and $d_{-1}$, 3 grid points for interpolating $K$, and we discretize aggregate and idiosyncratic productivity using 5 and 7 points, respectively. (We recompute the model using 9 points for aggregate productivity when computing impulse response functions). We construct a finer grid with 90 points for $a_{-1}$ and $d_{-1}$ to compute the final policy function used for simulation. Thus, our fine policy function must be solved for approximately 2 million grid points and their associated expectations. Our simulation uses 25,000 households for 5,000 periods with an initial burnin of 1,000 periods.

In order to simulate the recession shock, we first compute a discretization of the residuals estimated from an AR process for durable expenditures. (See Appendix 1). We then feed these residuals into the model as our aggregate productivity series and then hit the model with a simulated 4% reduction in capital in the period corresponding to 2008q4. While our Krusell-Smith approximation is highly accurate on average, it may be a poor approximation following this one-time shock. We address this by comparing forecast errors following the shock to forecast errors in general. Indeed we find that forecast errors are substantially worse immediately following the one-time shock, but they nevertheless remain highly accurate. Computing across various simulations, the maximum forecast error following the one-time shock is 0.1% of the capital shock, which translates into less than a 0.1% error in the forecast interest rate. This is substantially larger than the maximum .01% forecast errors arising without the

\[44\] Finer converge values didn't appear to affect the results.

45
shock, but is still extremely small. Exogenously raising and lower the interest rate by 0.1% only has tiny effects on simulated outcomes, so we view these errors as economically inconsequential.

10 Appendix 3: Estimating Durable Gaps

This appendix discusses the estimation of the empirical adjustment gaps and hazards using PSID microdata. The procedure begins by first estimating the relationship between variables in the structural model that have empirical counterparts and unobserved quantities of desired durable holdings. In the benchmark results, we estimate a linear regression of \( c, d_{-1}, a, \frac{c}{d_{-1}}, \frac{a}{d_{-1}+a} \) on \( d^*_\text{adjust} \) and \( d^*_\text{noadjust} \) (where \( d^*_\text{adjust} \) is the quantity of durables a household would choose today if its optimal policy is to pay the fixed cost and \( d^*_\text{noadjust} \) is the optimal policy if it chooses to not pay the fixed cost). We find that this specification has \( R^2 \) of over 0.99 so that these observables in the model are sufficient to explain desired durable holdings.\(^{45}\)

To estimate this procedure empirically, we must construct measures of these variables using PSID data. Beginning in 1999, the PSID contains detailed information on non-durable consumption, the value of housing and vehicles as well as various wealth holdings. Although more detailed non-durable consumption data is available beginning in 2003, for comparability we use only variables that are available beginning in 1999. The value for non-durable expenditures is the sum of all components of food consumption, utilities, transportation expenses, schooling expenses and health services. Our measure of \( d_{-1} \) is the sum of last periods housing value and vehicle values. Assets are the sum of business value, stocks, iras, cash, bonds, minus the value of outstanding debt. After constructing measures of \( c, d_{-1}, a \) per household member (with household head < age 65), we then deflate these nominal values using NIPA price indices and remove a household level fixed effect.

This then gives us an empirical counterpart to \( c, d_{-1}, a, \frac{c}{d_{-1}}, \frac{a}{d_{-1}+a} \) to which we apply the same structural relationship estimated from our model to generate empirical measures of \( d^*_\text{adjust,estimate} \) and \( d^*_\text{noadjust,estimate} \). The empirical gap in Figure 2, is defined as \( \log \left( \frac{d^*_\text{adjust,estimate}}{d^*_\text{noadjust,estimate}} \right) \).

\(^{45}\) Allowing for a more flexible functional form did not substantially enhance the predictive power. Similar results are also obtained by directly estimating the durable gap.
Given estimates of the durable gap, we can then calculate the empirical adjustment hazard as a function of the estimated durable gap. To compute the adjustment hazard, we must pick a threshold durable change that qualifies as an adjustment. The benchmark results use a threshold of a 50% change in the value of the durable stock to qualify as adjustment. This threshold generates an overall two-year adjustment probability of roughly 30%. Using different thresholds did not substantively affect our conclusions.

In addition to the results shown in Figure 2, we have also estimated bootstrapped confidence intervals for the average density and hazard to assess the statistical significance of the business cycle shifts that we estimate. Figure 13 shows estimated confidence intervals, which show that the time-variation we estimate is statistically significant.

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46 There is an indicator in the PSID for home sales, but it is not useful for 1999 because in that year, the question is phrased to cover a longer time span than it is in later years. Furthermore, this ignores changes in the value of vehicles.

47 With a bootstrap sample of 1000.